

EXPANDING AdS_5 BRANES: TIME DEPENDENT EIGENVALUE
PROBLEM AND PRODUCTION OF PARTICLESI. Brevik¹, M. Wold Lund, and G. RuøDepartment of Energy and Process Engineering, Norwegian University of
Science and Technology, N-7491 Trondheim, Norway**Abstract**

An analysis is first given of the situation where a scalar field is contained between two fixed, spatially flat, branes. The usual fine-tuning (RS) condition is relaxed, and the branes are allowed to possess a positive effective cosmological constant λ . We first analyze the eigenvalue problem for the Kaluza-Klein masses when the metric is time dependent, and consider in detail the case when λ is small. Thereafter we consider, in the case of one single brane, the opposite limit in which λ is large, acting in a brief period of time T , and present a "sudden approximation" calculation of the energy produced on the brane by the rapidly expanding de Sitter space during this period.

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1 INTRODUCTION

We begin by considering two static three-branes, situated at fixed positions $y = 0$ and $y = R$ in the transverse y direction. The branes are embedded in an AdS_5 space, and are subject to fine-tuning. This is the classic scenario of Randall and Sundrum [1]. There are by now several papers on brane cosmology in general [2]. If Λ (< 0) is the five-dimensional cosmological constant in the bulk, the Einstein equations are

$$R_{AB} - \frac{1}{2}g_{AB}R + g_{AB}\Lambda = \kappa^2 T_{AB}, \quad (1)$$

where $x^A = (t, x^1, x^2, x^3, y)$, and $\kappa^2 = 8\pi G_5$ is the gravitational coupling. On the first, $y = 0$ brane (the Planck brane) the tensile stress τ_0 is assumed positive. This is physically tantamount to assuming that the brane contains an ideal fluid whose equation of state is $p_0 = -\rho_0$, i.e., a "vacuum fluid", but being without any mechanical stress [3]. Thus τ_0 means physically the same as $-p_0$.

We introduce the effective four-dimensional cosmological constant on the two branes,

$$\lambda_0 = \frac{1}{6}\Lambda + \frac{1}{36}\kappa^4\tau_0^2, \quad \lambda_R = \frac{1}{6}\Lambda + \frac{1}{36}\kappa^4\tau_R^2. \quad (2)$$

If $\lambda_0 > 0$ then $\lambda_R > 0$ automatically [3]. Thus the system is two dS_4 branes embedded in an AdS_5 bulk.

We assume that the space is spatially flat, $k = 0$. The metric will be written as [4]

$$ds^2 = A^2(y)(-dt^2 + e^{2\sqrt{\lambda_0}t} \delta_{ik} dx^i dx^k) + dy^2, \quad (3)$$

with $\mu = \sqrt{-\Lambda/6}$, $H_0 = \sqrt{\lambda_0}$ being the Hubble constant on the first brane, and

$$A(y) = \frac{\sqrt{\lambda_0}}{\mu} \sinh[\mu(y_H - |y|)]. \quad (4)$$

Here y_H (> 0) is the horizon, determined by the relation $\sinh(\mu y_H) = \mu/\sqrt{\lambda_0}$. We assume $R < y_H$, so that the horizon at which $g_{tt} = 0$ does not occur in between the branes. Note that the metric (3) reduces to the RS metric [1] in the limit when $\lambda_0 = 0$.

Consider now a massive scalar field Φ in the bulk. Its action can in view of the Z_2 symmetry be written as

$$S = \frac{1}{2} \int d^4x \int_{-R}^R dy \sqrt{-G} (g^{AB} \partial_A \Phi \partial_B \Phi - M^2 \Phi), \quad (5)$$

with $G = \det(g_{AB})$.

In Ref. [5] we calculated the thermodynamic energy at finite temperatures, in the restrictive case of fine-tuning ($\lambda = 0$). Then, we could make use of the particle concept for the field without any conceptual problems. Our purpose in the present paper is to discuss certain aspects of the more complicated case when the four-dimensional cosmological constants λ_0 and λ_R are positive. In the

next section we review briefly for reference purposes the thermodynamic energy calculation in the fine-tuned case. In Sect. 3 we solve the eigenvalue problem for the scalar field, in principle. The calculation is carried out in full for the limiting case when λ_0 is small, $\sqrt{\lambda_0}/\mu \ll 1$. Somewhat surprisingly, in this limit the formalism does not allow any real value for the Kaluza-Klein masses m_n . Finally, in Sect. 4 we consider the opposite limit in which λ is large, but active during a brief period of time T only. We here restrict ourselves to the case of one single brane. This situation means physically a rapid expansion of the de Sitter space, from an initial static (fine-tuned) case I to another static case II. The situation is tractable analytically when one makes use of the "sudden" approximation in quantum mechanics. The production of particles is estimated from use of the Bogoliubov transformation, relating the states I and II. The method was first made use of in a cosmological context by Parker [6].

2 ON THE FINE-TUNED CASE

We assume the branes to be spatially flat, $k = 0$, and also to be fine-tuned, $\lambda_0 = \lambda_R = 0$. As mentioned above, we can here make use of the particle picture. We assume a finite temperature, $T = 1/\beta$, and start by noting that the free energy F for a bosonic scalar field in a three-dimensional volume V is given by [5]

$$\beta F = -\ln Z = V \int \frac{d^3 p}{(2\pi)^3} \ln \left[2 \sinh\left(\frac{1}{2}\beta E_p\right) \right], \quad (6)$$

where Z is the partition function and $E_p = \sqrt{\mathbf{p}^2 + M^2}$ the particle energy.

The metric is now

$$ds^2 = e^{-2\mu|y|} \eta_{\alpha\beta} dx^\alpha dx^\beta + dy^2. \quad (7)$$

We allow for a boundary mass term by letting $M \rightarrow \bar{M}$, where

$$\bar{M}^2 = M^2 + 2b\mu[\delta(y) - \delta(y - R)], \quad (8)$$

b being a constant [7]. The field equation $(\square - M^2)\Phi = 0$ in the bulk takes the form

$$\left[e^{2\mu|y|} \eta^{\alpha\beta} \partial_\alpha \partial_\beta + e^{4\mu|y|} \partial_y (e^{-4\mu|y|} \partial_y) - M^2 \right] \Phi(x^\alpha, y) = 0. \quad (9)$$

The Kaluza-Klein decomposition

$$\Phi(x^\alpha, y) = \frac{1}{\sqrt{2R}} \sum_{n=0}^{\infty} \Phi_n(x^\alpha) f_n(y) \quad (10)$$

yields the following equation for the KK masses m_n :

$$e^{-2\mu|y|} \left[-f_n''(y) + 4 \operatorname{sign}(y) \mu f_n'(y) + M^2 f_n(y) \right] = m_n^2 f_n(y), \quad (11)$$

whose solution (here given for $0 < y < R$) can be written as

$$f_n(y) = \frac{e^{2\mu y}}{N_n} \left[J_\nu \left(x_n e^{\mu(y-R)} \right) + b_n Y_\nu \left(x_n e^{\mu(y-R)} \right) \right], \quad (12)$$

with $x_n = m_n/(a\mu)$, $a = e^{-\mu R}$, $\nu = \sqrt{4 + M^2/\mu^2}$. The boundary conditions are found by integrating Eqs. (11) across the branes. The field may be either even (untwisted) or odd (twisted) under the Z_2 symmetry. We define the altered Bessel functions $j_\nu(z) = (2-b)J_\nu(z) + zJ'_\nu(z)$, $y_\nu(z) = (2-b)Y_\nu(z) + zY'_\nu(z)$, and consider here only the even case, $f_n(y) = f_n(-y)$. The KK masses are given as roots of the equation

$$D(x_n) \equiv j_\nu(x_n)y_\nu(ax_n) - j_\nu(ax_n)y_\nu(x_n) = 0, \quad (13)$$

and the coefficient b_n in Eq. (12) is $b_n = -j_\nu(x_n)/y_\nu(n_n)$.

In the physically reasonable limit of $m_n \ll \mu$ and $a \ll 1$ we get

$$x_n = (n + \frac{\nu}{2} - \frac{3}{4})\pi, \quad n = 1, 2, \dots, \quad (14)$$

the approximation being better the larger the value of n . The quantities x_n are thus for low n of order unity. At finite temperatures the bosonic free KK energy F^{KK} can be expressed as a contour integral in the following form:

$$\beta F^{KK} = V \int \frac{d^3p}{(2\pi)^3} \frac{i}{2\pi} \int_C dx \frac{d}{dx} \ln \left[2 \sinh \left(\frac{1}{2} \beta \sqrt{\mathbf{p}^2 + a^2 \mu^2 x^2} \right) \right] \ln D(x) \quad (15)$$

for the even modes, the contour C encompassing the zero points for $D(x)$ [5].

This concludes our brief review of the $k = 0$, $\lambda = 0$ theory, and we now proceed to consider the non-static case for which λ is different from zero.

3 TWO BRANES, WHEN $\lambda > 0$

We shall still keep the spatial curvature k equal to zero, but assume now that $\lambda_0 > 0$ which, as noted above, implies that also $\lambda_R > 0$. The metric is given by Eqs. (3) and (4). To begin with, we do not restrict λ_0 to be small.

The presence of curved branes (meaning here the time varying scale factor $a = e^{H_0 t}$), complicates the calculation of the vacuum energy in two ways. First, it results in a non-trivial spectrum of the KK excitations on the brane. Secondly, for each given excitation the vacuum energy becomes more complicated because of the curvature. Actually, the calculation of the determinant of the relevant differential operator is complicated to such an extent that its evaluation becomes rather intractable. For this reason, mostly conformal fields have been investigated so far. Some papers are listed in Ref. [8]; they are dealing with conformal and partly also with massive fields. Below, we will focus attention on one specific issue, namely how to find the energy eigenvalues of the massive scalar field.

The y equation giving the KK masses now takes the form

$$-f''_n(y) + 4 \operatorname{sign}(y) \mu \coth[\mu(y - y_H)] f'_n(y) + M^2 f_n(y) = \frac{m_n^2 \mu^2}{\lambda \sinh^2[\mu(y - y_H)]} f_n(y), \quad (16)$$

for which the solutions are seen to be either even or odd. We consider here the solution for $y > 0$. Introducing the variable z by $z = \cosh \mu(y_H - y)$, we can write the solution as

$$f_n(z) = \frac{1}{N_n(z^2 - 1)^{3/4}} (P_\alpha^\beta(z) + c_n Q_\alpha^\beta(z)), \quad (17)$$

where P_α^β and Q_α^β are associated Legendre polynomials, $\alpha = \sqrt{4 + M^2/\mu^2} - 1/2$, $\beta = \sqrt{9/4 - m_n^2/\lambda}$, and N_n is a normalization constant. Note that when y increases from 0 to R , z decreases from z_0 to z_R , where $z_0 = \sqrt{1 + \mu^2/\lambda_0}$, $z_R = \cosh[\mu(y_H - R)]$. The Legendre polynomials are usually taken to converge for $|z| < 1$, but can be analytically continued to $|z| > 1$, which is the region of interest here. (To make the functions single valued, a cut is made along $-\infty < z < 1$.)

For *odd* solutions we have as boundary conditions

$$f_n|_{z_0, z_R} = 0, \quad (18)$$

so that the coefficients c_n in Eq. (17) are determined as

$$c_n = -P_\alpha^\beta(z_0)/Q_\alpha^\beta(z_0) = -P_\alpha^\beta(z_R)/Q_\alpha^\beta(z_R), \quad (19)$$

and the KK masses are given by

$$P_\alpha^\beta(z_0)Q_\alpha^\beta(z_R) - P_\alpha^\beta(z_R)Q_\alpha^\beta(z_0) = 0. \quad (20)$$

Note that the KK masses enter through the upper index in the associated Legendre functions.

For *even* solutions the boundary conditions are likewise found by integrating Eq. (16) across the branes. Let us introduce the notation

$$\begin{aligned} p_\alpha^\beta(z) &= \frac{1}{z^2 - 1} \left(\frac{3}{2}z + b \right) P_\alpha^\beta(z) + (P_\alpha^\beta(z))', \\ q_\alpha^\beta(z) &= \frac{1}{z^2 - 1} \left(\frac{3}{2}z + b \right) Q_\alpha^\beta(z) + (Q_\alpha^\beta(z))'. \end{aligned} \quad (21)$$

Then, a brief calculation shows that the boundary conditions yield for the coefficients c_n

$$c_n = -p_\alpha^\beta(z_0)/q_\alpha^\beta(z_0) = -p_\alpha^\beta(z_R)/q_\alpha^\beta(z_R), \quad (22)$$

and the corresponding KK masses are given by

$$p_\alpha^\beta(z_0)q_\alpha^\beta(z_R) - p_\alpha^\beta(z_R)q_\alpha^\beta(z_0) = 0. \quad (23)$$

We outline how the KK masses m_n can be calculated, when we start from the natural assumption that $\{\mu, \lambda_0, R\}$ are known input parameters. The horizon is first determined from the equation $\sinh(\mu y_H) = \mu/\sqrt{\lambda_0}$, and z_0 and z_R are found from the expressions above. If moreover the scalar mass M is known, the

value of α follows, and the KK masses finally follow by solving Eqs. (20) and (23) in the odd and even cases, respectively.

Due to the appearance of m_n in the upper index β , the formalism becomes however quite complicated. We shall not here carry on the analysis further, except from pointing out the following and perhaps surprising behavior of the formalism in the limiting case of a small- λ_0 , narrow-gap situation. Let us first assume that $\lambda_0/\mu \ll 1$. Then, $z_0 = \sqrt{1 + \mu^2/\lambda_0} = \cosh(\mu y_H) \gg 1$. Moreover, if we take the gap width R to be small compared with the horizon, $R \ll y_H$, then we can also assume that $z_R = \cosh[\mu(y_H - R)] \gg 1$. It becomes thus natural to make use of the following approximate expressions for the analytically continued Legendre functions, valid when $|z| \gg 1$:

$$P_\alpha^\beta(z) = \left\{ \frac{2^\alpha \Gamma(\alpha + \frac{1}{2})}{\sqrt{\pi} \Gamma(\alpha - \beta + 1)} z^\alpha + \frac{\Gamma(-\alpha - \frac{1}{2})}{2^{\alpha+1} \sqrt{\pi} \Gamma(-\alpha - \beta)} z^{-\alpha-1} \right\} \left(1 + O\left(\frac{1}{z^2}\right) \right). \quad (24)$$

This expression holds when $2\alpha \neq \pm 1, \pm 3, \dots$. Similarly,

$$Q_\alpha^\beta(z) = \sqrt{\pi} \frac{e^{\beta\pi i}}{2^{\alpha+1}} \frac{\Gamma(\alpha + \beta + 1)}{\Gamma(\alpha + \frac{3}{2})} z^{-\alpha-1} \left(1 + O\left(\frac{1}{z^2}\right) \right), \quad (25)$$

which holds when $2\alpha \neq -3, -5, \dots$ [9]. In the case of odd solutions, insertion of Eqs. (24) and (25) in Eq. (20) leads to the condition $1/\Gamma(\alpha - \beta + 1) = 0$, which means $\alpha - \beta + 1 = -n$ with $n = 0, 1, 2, \dots$. In other words

$$\sqrt{\frac{9}{4} - \frac{m_n^2}{\lambda}} = n + \frac{1}{2} + \sqrt{4 + \frac{M^2}{\mu^2}}. \quad (26)$$

This is a condition, however, that cannot be satisfied for any $n \geq 0$. The case of even solutions leads to the same conclusion. Thus in the limit of small λ_0 and narrow gap R there is no physical solution for m_n .

4 ENERGY PRODUCTION CALCULATED VIA THE BOGOLIUBOV TRANSFORMATION

Instead of dealing with the complicated case of a time dependent metric, it is sometimes possible to proceed in another way which is both mathematically simple and physically instructive. The method applies in cases where the change of the system takes place abruptly (corresponding to the "sudden approximation" in quantum mechanics). The method implies use of the Bogoliubov transformation relating two vacua, designated in the following by I (in) and II (out). As mentioned in Sect. 1, in a cosmological context the method was introduced by Parker [10], and it has been made use of later, for instance in connection with the formation of cosmic strings [10, 11].

We assume now that there is only one single brane, situated at $y = 0$, As before, we take $k = 0, \lambda_0 > 0$, and we start from the state I in which the

metric is static and the particle concept for the field thus directly applicable. This state corresponds to times $t < 0$. The line element on the brane at these times is thus $ds_I^2 = -dt^2 + \delta_{ik}dx^i dx^k$. During the time period $0 < t < T$ the de Sitter metric on the brane is rapidly expanding, and the line element is $ds^2 = -dt^2 + e^{2H_0 t} \delta_{ik}dx^i dx^k$. For $t > T$ (state II) the metric is again assumed static, so that $ds_{II}^2 = -dt^2 + \alpha^2 \delta_{ik}dx^i dx^k$, with $\alpha = e^{H_0 T}$. For state II the particle concept is thus again applicable. The vacua corresponding to states I and II are denoted by $|0, I\rangle$ and $|0, II\rangle$.

The field equation on the brane reads, in both static cases,

$$\ddot{\Phi}_n - \partial^i \partial_i \Phi_n + m_n^2 \Phi_n = 0, \quad (27)$$

and in state I the basic modes can be written

$$u_{\mathbf{k}I} = \frac{(2\pi)^{-3/2}}{\sqrt{2\omega_I}} \exp(i \mathbf{k}_I \cdot \mathbf{x} - i\omega_I t), \quad (28)$$

with $\mathbf{k}_I = (k_1, k_2, k_3)_I$. The dispersion relation is $\omega_I^2 = \mathbf{k}_I^2 + m_n^2$. In state II the basic modes $u_{\mathbf{k}II}$ are given by the same expression, only with the replacements $\omega_I \rightarrow \omega_{II}$, $\mathbf{k}_I \rightarrow \mathbf{k}_{II}$. Here $\mathbf{k}_{II} = (k_1, k_2, k_3)_{II}$ and $\omega_{II}^2 = \alpha^{-2} \mathbf{k}_{II}^2 + m_n^2$ (note that $k_{II}^i k_{II}^i = \alpha^{-2} k_{II}^i k_{II}^i = \alpha^{-2} \mathbf{k}_{II}^2$). In both cases the mode functions satisfy the normalization condition

$$(u_{\mathbf{k}}, u_{\mathbf{k}'}) \equiv -i \int u_{\mathbf{k}}(x) \overleftrightarrow{\partial}_0 u_{\mathbf{k}'}(x) d^3x = \delta(\mathbf{k} - \mathbf{k}'), \quad (29)$$

the other products vanishing, as usual. We consider the same positions in space before and after the expansion, i.e., $x_I^\mu = x_{II}^\mu \equiv x^\mu$. Correspondingly, we identify the covariant components of the wave vectors, $k_i^I = k_i^{II} \equiv k_i$. The dispersion relations then become

$$\omega_I^2 = \mathbf{k}^2 + m_n^2, \quad \omega_{II}^2 = \alpha^{-2} \mathbf{k}^2 + m_n^2. \quad (30)$$

We expand Φ_n in either I or II modes,

$$\Phi_n(x) = \int \frac{d^3k}{(2\pi)^3} \left(a_{\mathbf{k}} u_{\mathbf{k}}(x) + a_{\mathbf{k}}^\dagger u_{\mathbf{k}}^*(x) \right), \quad (31)$$

and relate the two modes via a Bogoliubov transformation [12]

$$u_{\mathbf{k}II} = \int \frac{d^3k'}{(2\pi)^3} (\alpha_{\mathbf{k}\mathbf{k}'} u_{\mathbf{k}'I} + \beta_{\mathbf{k}\mathbf{k}'} u_{\mathbf{k}'I}^*), \quad (32)$$

implying for the operators

$$a_{\mathbf{k}I} = \int \frac{d^3k'}{(2\pi)^3} (\alpha_{\mathbf{k}'\mathbf{k}} a_{\mathbf{k}'II} + \beta_{\mathbf{k}'\mathbf{k}}^* a_{\mathbf{k}'II}^\dagger). \quad (33)$$

The number of particles $N(k)$ in \mathbf{k} space in state II, produced by the rapid expansion, is

$$N(k) = \langle 0, II | a_{\mathbf{k}I}^\dagger a_{\mathbf{k}I} | 0, II \rangle = \int \frac{d^3k'}{(2\pi)^3} |\beta_{\mathbf{k}'\mathbf{k}}|^2, \quad (34)$$

where we calculate

$$\beta_{\mathbf{k}'\mathbf{k}} = -(u_{\mathbf{k}'II}, u_{\mathbf{k}I}^*) = \frac{1}{2} \left(\sqrt{\frac{\omega_I}{\omega_{II}}} - \sqrt{\frac{\omega_{II}}{\omega_I}} \right) \delta(\mathbf{k} + \mathbf{k}'). \quad (35)$$

We insert this into Eq. (34), and make use of the effective substitutions

$$\int \frac{d^3k'}{(2\pi)^3} [(2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')]^2 \rightarrow (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')|_{\mathbf{k}' \rightarrow -\mathbf{k}} \rightarrow V, \quad (36)$$

where V is the volume, to get

$$N(k) = \frac{V}{4(2\pi)^3} \left(\frac{\omega_I}{\omega_{II}} + \frac{\omega_{II}}{\omega_I} - 2 \right). \quad (37)$$

Multiplying with ω_{II} to get the energy of each mode, we find by integrating over all \mathbf{k} the following simple expression for the produced energy:

$$E = \pi V \int_0^K \omega_{II} \left(\frac{\omega_I}{\omega_{II}} + \frac{\omega_{II}}{\omega_I} - 2 \right) k^2 dk. \quad (38)$$

Here K is introduced as an upper limit, to prevent the UV divergence arising from the idealized sudden approximation.

We shall consider only one limit of this expression. As noted in Sect. 2, $x_n = (m_n/\mu)e^{\mu R} \sim 1$ usually, implying that $m_n/\mu \ll 1$. Let us simply assume here that $m_n = 0$, so that $\omega_I/\omega_{II} = \alpha$ according to Eq. (30). Then, Eq. (38) yields

$$E = \frac{1}{4} \pi V K^4 \left(1 + \frac{1}{\alpha^2} - \frac{2}{\alpha} \right). \quad (39)$$

To make a rough estimate of the energy produced we may, following Parker [10], take into account that the process actually takes some finite time Δt . The consequence is that the spectrum falls rapidly for frequencies much larger than $1/\Delta t$. Let us simply assume that K is of the same order of magnitude as $1/\Delta t$. Then, Eq. (39) yields in dimensional units

$$E \sim \frac{\pi \hbar V}{4c^3 (\Delta t)^4} \left(1 + \frac{1}{\alpha^2} - \frac{2}{\alpha} \right). \quad (40)$$

If we take α to be of order unity, and set $\Delta t \sim t \sim 10^{-33}$ s (i.e., inflationary times), then this expression yields $E/V \sim 10^{73}$ ergs/cm³. For the sake of comparison, we note that this is about 13 orders of magnitude less than the energy density associated with the formation of a cosmic string [10].

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